

**AMPERE'S LAW :**

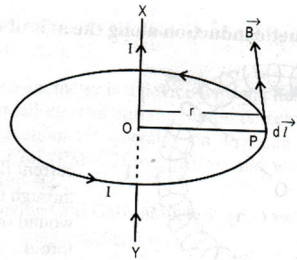
The line integral of magnetic field of induction  $\vec{B}$  around any closed path in free space is equal to absolute permeability of free space ( $\mu_0$ ) times the total current flowing through area bounded by the path.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

In Ampere's law, we imagine an Amperian loop i.e. a closed curve around a current carrying conductor. Further we imagine the loop (selected according to our convenience) to be made up of large number of small elements each of length  $dl$ . Then we determine the scalar products of  $\vec{B}$  and  $d\vec{l}$  for each element and add all such products for entire loop. The direction along which loop is traced is the direction of element of length  $d\vec{l}$   
 $\oint \vec{B} \cdot d\vec{l} = \oint B \cdot dl \cos\theta$ , where  $\theta$  is the angle between  $\vec{B}$  and  $d\vec{l}$

**MAGNETIC FIELD DUE TO A LONG STRAIGHT CONDUCTOR CARRYING CURRENT:**

Consider as infinitely long straight conductor XY carrying an electric current  $I$ . Let P be a point at a distance  $r$  from the conductor. We have to determine the magnetic induction of magnetic field at P due to current flowing through the conductor.



Let us choose an Amperian loop as an imaginary circle of radius  $r$  (perpendicular to straight conductor).

$\vec{B}$  : magnetic induction at P, due to current  $I$  flowing through the conductor.

$d\vec{l}$ : length of small element of circle around the wire

According to Ampere's law,

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$ , but  $\oint \vec{B} \cdot d\vec{l} = \oint B \cdot dl \cos\theta$ , where  $\theta$  is the angle between  $\vec{B}$  and  $d\vec{l}$  which is zero. Thus,

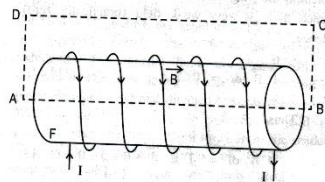
$$\oint B \cdot dl = \mu_0 I \quad \text{Therefore, } B \oint dl = \mu_0 I$$

$$B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0}{4\pi} \frac{2I}{r}$$

**Magnetic Induction along the axis of a long straight solenoid:**

Let  $n$ : number of turns per unit length  
 $I$ : current sent through it, due to which magnetic field is created.  
 Consider a rectangular path ABCD. Let  $AB=L$ . Hence,  $nL$  is the number of turns enclosed by the rectangle ABCD.



Total current flowing =  $nLI$

$\vec{B}$ : Magnetic induction at a point well inside the solenoid.

According to Ampere's law,  $\oint \vec{B} \cdot d\vec{l} = \mu_0(nLI)$ . But for ABCDA,

$$\oint \vec{B} \cdot d\vec{l} = \int_A^B \vec{B} \cdot d\vec{l} + \int_B^C \vec{B} \cdot d\vec{l} + \int_C^D \vec{B} \cdot d\vec{l} + \int_D^A \vec{B} \cdot d\vec{l}$$

$$\int_B^C \vec{B} \cdot d\vec{l} = \int_C^D \vec{B} \cdot d\vec{l} = 0, \text{ since } \vec{B} \text{ is perpendicular to } BC \text{ and } AD$$

$$\int_C^D \vec{B} \cdot d\vec{l} = 0,$$

Since outside the solenoid the magnetic field lines are widely spaced hence there is a very weak field (practically zero)

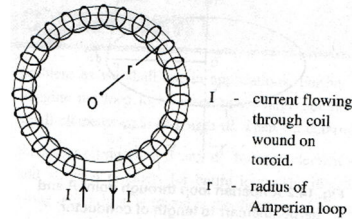
$$\text{Thus, } \oint \vec{B} \cdot d\vec{l} = \int_A^B \vec{B} \cdot d\vec{l} = \int_A^B B dl \cos\theta = \int_A^B B dl \cos 0 = \int_A^B B dl$$

$$\int_A^B B dl = B \cdot L$$

Thus,  $\oint \vec{B} \cdot d\vec{l} = \mu_0(nLI)$  becomes  $B \cdot L = \mu_0(nLI)$ . Therefore,  $B = \mu_0(nI)$

Near the ends,  $B = \mu_0(nI)/2$

**MAGNETIC INDUCTION ALONG THE AXIS OF TOROID:**



Toroid is a solenoid bent into a shape of a hollow donut.

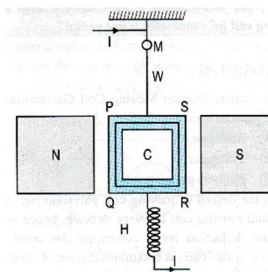
Consider a Amperian loop of radius  $r$ . According to Ampere's law,  $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$ , but here the total current flowing is  $NI$

where  $N$  is the total number of turns. Thus,  $\oint \vec{B} \cdot d\vec{l} = \mu_0 NI$ . Now,  $\vec{B}$  and  $d\vec{l}$  are in same direction. Thus  $\oint \vec{B} \cdot d\vec{l} = B \cdot dl = B(2\pi r)$

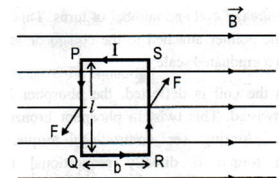
$$\text{Hence, } B(2\pi r) = \mu_0 NI \quad \text{Thus, } B = \frac{\mu_0 NI}{2\pi r} = \mu_0 nI$$

where  $n$  = number of turns per unit length of toroid. =  $\frac{N}{2\pi r}$

**MCG**



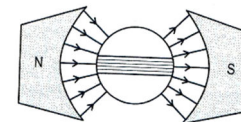
**For CONSTRUCTION**  
Please refer TEXTBOOK



There is not force acting on PS and QR, because they are parallel to  $\vec{B}$   
 Moment of a couple or torque = Magnitude of one force  $\times$  perpendicular distance between the forces

$$\tau = (nBil)(b) = BinA, \text{ where } A = l \times b$$

The above torque deflects the coil, hence is called deflecting torque



Since the magnetic field is radial, the deflecting torque in all positions of the coil is the same and is equal to  $BinA$ . Thus, the deflecting torque is proportional to the number of turns, magnetic induction of the magnetic field, current flowing in the coil and area of the coil.

As the coil is deflected, the phosphor bronze spring gets twisted creating a restoring torque which is directly proportional to the deflection  $\theta$ .

$$\tau_c \propto \theta, \quad \tau_c = c\theta, \text{ where } c = \text{twist constant or restoring torque per unit twist}$$

For equilibrium,  $\tau_d = \tau_c$  i.e.  $BinA = c\theta$

$$i = \theta \left( \frac{c}{nAB} \right). \text{ Hence, } i = K\theta \quad i \propto \theta$$

Thus current flowing through MCG is directly proportional to the angle of deflection of the coil.

**Advantages:**

- not affected by strong magnetic field
- High torque/weight ratio
- Very accurate and reliable
- Uniform scale

**Disadvantage:**

- Change in temperature affects restoring torque
- Restoring torque cannot be easily changed
- Possible damage to the phosphor bronze suspension due to usage stress
- Cannot be used for a.c. measurement

## SENSITIVITY OF MCG

A MCG is said to be sensitive if it gives larger change in deflection for smaller changes of current flowing through it.

Let  $I$  be the initial current and  $I+dI$  the final current and  $\theta$  and  $\theta+d\theta$  the corresponding initial and final angle of deflection of the MCG. Then Sensitivity  $S = d\theta/dI$

The current flowing through MCG is

$$I = \left(\frac{c}{nBA}\right) \theta$$

Differentiating,

$$dI = \left(\frac{c}{nBA}\right) d\theta$$

$$S = \frac{d\theta}{dI} = \left(\frac{nBA}{c}\right)$$

Thus sensitivity of a galvanometer can be increased by

- Increasing the number of turns ( $n$ )
- Increase the magnetic Induction ( $B$ ), by using stronger magnet
- Increase the area of coil ( $A$ )
- Decreasing the restoring torque per unit displacement ( $c$ )

## ACCURACY OF MCG

For more accuracy, the relative error in the measurement of current should be less.

Let  $\theta$  be the true value of deflection for the current  $I$  and let  $d\theta$  and  $dI$  be the error in measuring the same. Then,

$$I = \left(\frac{c}{nBA}\right) \theta$$

Differentiating,

$$dI = \left(\frac{c}{nBA}\right) d\theta$$

$$\frac{dI}{I} = \left(\frac{d\theta}{\theta}\right)$$

If the fractional error  $dI/I$  is small the MCG is said to be more accurate and this is possible for large value of  $\theta$  for a given  $d\theta$

## AMMETER

A Permanent Magnet Moving Coil (PMMC) cannot directly be used as an Ammeter due to the following:

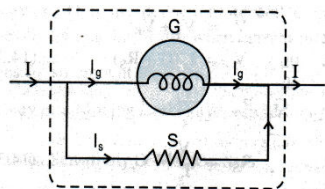
>>Its Resistance is not small enough

>>Its current range for full scale deflection is small

Thus, we shunt the galvanometer (using a low value resistance in parallel) and the new device is calibrated to read the current directly.

Shunting allows most current to pass through the shunt and a very negligible current pass through the galvanometer. Shunt is made up of manganin, since it has a very negligible temperature coefficient of resistance.

Ideal ammeter should have ZERO resistance. Practical Ammeter have finite low resistance.



$I$  is the maximum current to be measured and  $I_g$  is the current flowing through the galvanometer for full scale deflection. ' $I_s$ ' is the shunt current

$$I_s = I - I_g$$

G: Galvanometer resistance

S: Shunt resistance.

Voltage across G = Voltage across S

$$I_g G = I_s \cdot S$$

Thus,  $I_g G = (I - I_g)S$

$$\text{Thus, } S = \left(\frac{I_g}{I - I_g}\right) G$$

This is the resistance we need to connect in parallel to convert the galvanometer into an Ammeter capable of measuring current upto  $I$ .

## Functions of Shunt resistor (S):

- Reduces the effective resistance of the galvanometer
- Increases the range of the instrument
- Provides an alternative path for the current to pass thus protecting the galvanometer

## VOLTMETER

A Permanent Magnet Moving Coil (PMMC) cannot directly be used as an Voltmeter due to the following:

>>Its Resistance is not high hence connecting it across high potential will cause high currents, thus burning the coil of the MCG.

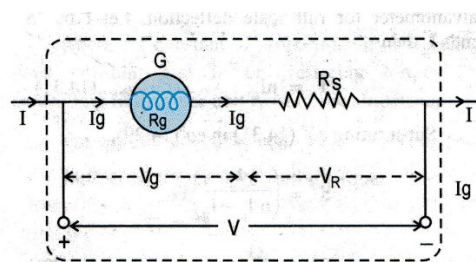
>>Its voltage range for full scale deflection is small

A PMMC instrument in series with high resistance can be used as voltmeter.

Here too, manganin (alloy of copper, manganese and nickel) is used since it has a negligible temperature coefficient of resistance.

Voltmeter is always connected in parallel across the device whose potential difference is to be measured.

Ideal voltmeter has infinite resistance, hence when attached to the device is parallel will not affect the current flowing in that device.



Let,  $I_g$  = Current in galvanometer for full scale deflection

G: Galvanometer Resistance       $R_s$ : High value series resistance

V: Potential Difference to be measured.

$$V = V_g + V_r = I_g \cdot G + I_g \cdot R_s = I_g(G + R_s)$$

## Functions of high series resistor (Rs):

- Increases the effective resistance of the galvanometer
- Increases the range of the instrument
- Protecting the galvanometer from damage due to large current

**FOR CYCLOTRON REFER TEXTBOOK / NOTES ON EXPERIMENTS**